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Light Distribution Near the Line Focus in a HSURIA Laser Resonator

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The intensity distribution about the line focus inside a HSURIA resonator is obtained theoretically and used to study the gas-breakdown problem for high-energy lasers.

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CONTENTS

LIGHT DISTRIBUTION NEAR THE LINE FOCUS IN A HSURIA LASER RESONATOR

INTRODUCTION

The half-spherical unstable resonator with internal axicon (HSURIA) has been developed for use with HF and DF lasers [1]. It appears to be particularly well suited to high-energy chemical lasers of this type chiefly because of its accommodation of a thin cylindrical gain medium, its high output coupling, and its relatively compact optical configuration. The HSURIA does present some serious difficulties, however. First, it appears highly doubtful that the HSURIA can contain a simple low-order optical mode which produces the preferred output beam with a Gaussian intensity profile [2]. Second, it appears that for high-energy systems there is a gas-breakdown problem for operation at atmospheric pressure. It is this second problem which is addressed in the present report.

The gas-breakdown problem results from the use of the conical reflector at the back of the annular gain region as shown in Fig. 1. The most basic HSURIA resonator as shown in the figure consists, from left to right, of a convex spherical front mirror, an axicon combined with an conical reflector (sometimes called a reflaxicon), the annular gain medium, and a second conical reflector which serves as the back mirror for the resonator. The annular gain medium results from constraints placed on the laser system by the present design of the gas-flow system for the HF and DF lasers and appears to be unavoidable, at least for the present. The reflaxicon and conical back reflector are used to accommodate this restriction on gain-medium geometry. It does not appear possible to replace the troublesome back reflector by a plane mirror. This results in high alignment sensitivity. In experiments with HSURIA resonators it is nearly impossible to align and hold such a mirror in place accurately enough to achieve a steady mode [3]. The conical back mirror shown in the figure solves this problem nicely but introduces a new problem, the gas-breakdown problem considered here.

The conical back reflector removes the alignment sensitivity, but it brings the field inside of the resonator to a diffraction-limited line focus at which all of the energy is concentrated along a line a few centimeters long. Clearly for sufficiently high energy systems such a focusing of the field inside of the resonator will cause gas breakdown, prevent proper operation, and possibly damage laser components. Thus the output power will be severely limited, at least for lasers operated with the resonator containing a gas at atmospheric pressure.

In this report a theoretical study is presented in which the fields associated with the line focus are derived, the maximum laser output beam energy is calculated for operation at atmospheric pressure, and possible solutions to this problem are discussed.

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where

$$m = \sqrt{1 - p^2 - q^2}. \tag{3}$$

By substituting Eq. (1) into (2), we have

$$\psi(\vec{x}) = \frac{2\sqrt{I_0}\,\sigma}{\sqrt{2\pi}\,\lambda} \iint\limits_{p^2+q^2\leqslant 1} \frac{1}{m} e^{ikxp} \cos(kmz)\, dp\, e^{-k^2\sigma^2 q^2/2} e^{ikqy}\, dq. \tag{4}$$

We note that if we assume σ/λ is large, the kernel of the integral over q contributes principally only in the neighborhood where $q \approx 0$. This approximation will be justified later. Thus we can substitute the asymptotic approximation to Eq. (3), namely,

$$m \underset{q\to 0}{\sim} \sqrt{1 - p^2} + O(q^2), \tag{5}$$

into Eq. (4) to obtain

$$\psi(\vec{x}) \underset{\sigma/\lambda\to\infty}{\sim} \frac{2\sqrt{I_0}\,\sigma}{\sqrt{2\pi}\,\lambda} \int_{-1}^{1} e^{-k^2\sigma^2q^2/2} e^{ikqy}\, dq \int_{-1}^{1} \frac{1}{\sqrt{1-p^2}}\, e^{ikxp} \cos\left(k\sqrt{1-p^2}\, z\right) dp, \tag{6}$$

so that the integrals become independent. The first integral in Eq. (6) is tabulated (as in Ref. 8, p. 85, Eq. (710.0)). Evaluating the first integral, while making the transformation of variables

$$p = \cos\theta,\ \sqrt{1-p^2} = \sin\theta,\ dp = -\sin\theta\, d\theta, \tag{7}$$

we obtain

$$\psi(\vec{x}) \underset{\sigma/\lambda\to\infty}{\sim} \frac{\sqrt{I_0}}{2\pi}\, e^{-y^2/2\sigma^2} \int_0^{\pi/2} 4\cos(kx\cos\theta)\cos(kz\sin\theta)\, d\theta. \tag{8}$$

The integral in Eq. (8) is evaluated in the Appendix, which gives

$$\int_0^{\pi/2} \cos(kx\cos\theta)\cos(kz\sin\theta)\,d\theta = \frac{\pi}{2} J_0(k\sqrt{x^2+z^2}). \tag{9}$$

By substituting (9) into (8) we have

$$\psi(\vec{x}) \underset{\sigma/\lambda\to\infty}{\sim} \sqrt{I_0}\, e^{-y^2/2\sigma^2} J_0(k\sqrt{x^2+z^2}), \tag{10}$$

which is the field amplitude over the focal region. (Equation (34) of Ref. 9 is a similar result found in a different manner.)

Since it is the intensity of the field at focus which causes the gas-breakdown problem, we use Eq. (10) to calculate the intensity as

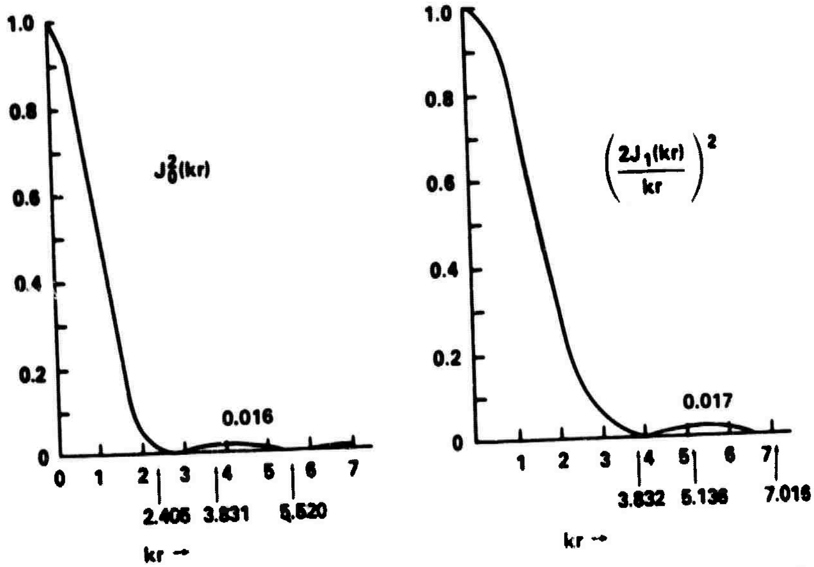
$$I(\vec{x}) = |\psi(\vec{x})|^2 = I_0\, e^{-y^2/\sigma^2} J_0^2(k\sqrt{x^2+z^2}). \tag{11}$$

We can now justify the asympototic approximation made in Eq. (6) that $\sigma/\lambda \to \infty$. From Eq. (11) we see that σ/λ represents the length of the focal line in wavelengths, which for all practical situations is a large number (of the order of 10^5).

We note that the intensity over the line focus is different from the well-known Airy disk pattern

$$I(\vec{x}) = I_0 \left[\frac{2J_1(k\alpha r)}{k\alpha r}\right]^2, \tag{12}$$

which represents the intensity distribution about the point focus formed by a lens with an angular aperture α [10, section 8.5.2]. Comparison of these two functions in Fig. 2 for the optimum case ($\alpha = 1$) indicates that the line focus is slightly sharper but otherwise apparently similar. However, a calculation of the energy contained within a cylinder of radius a centered on the focal line indicates that the line focus as given by Eq. (11) has unusual properties.



(a) Line focus

(b) Point focus [10, section 8.5.2]

Fig. 2 — Comparison of the radial dependence of two focal field intensities

We obtain the energy contained over the line focus inside a cylinder of radius a centered on the y axis by simply integrating the intensity over the interior of the cylinder:

$$\epsilon = \int_0^a \int_0^{2\pi} \int_{-\infty}^{\infty} I(\vec{x}) r \, dy d\theta dr. \tag{13}$$

By substituting Eq. (11) into (13) and carrying out the integration using a tabulated integral [11, section 11.3.34], we find that

$$\epsilon = \pi^{3/2} \sigma a^2 I_0 \left[J_0^2(ka) + J_1^2(ka) \right] . \tag{14}$$

For larger ka, so that the radius of the cylinder is much larger than the wavelength of the radiation, we have

$$\epsilon \sim 2\sqrt{\pi} \frac{\sigma I_0}{k^2} ka, \tag{15}$$

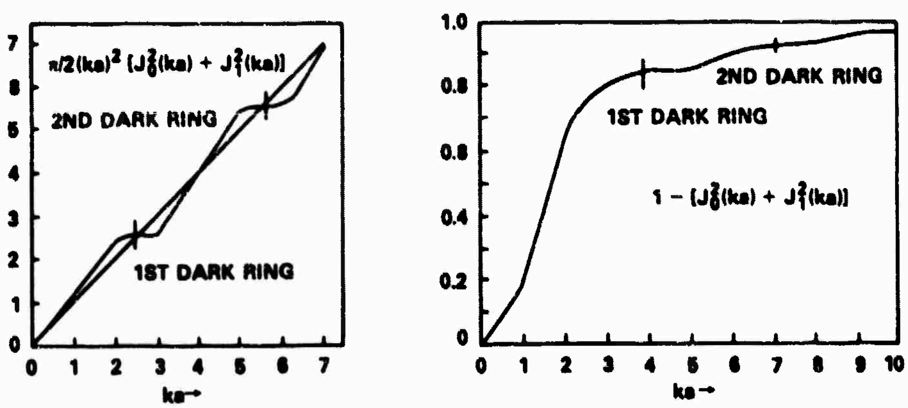
where we have used the asymptotic form for the Bessel functions [11, section 9.2.1]. From Eq. (15) we see that the energy increases linearly with radius a. Thus the energy is not restricted to the region near the focal line but extends away from it even into the far field. This is strikingly different from the conventional point focus. In Fig. 3 the energy contained within a circle of radius a about a conventional point focus as given by Born and Wolf [10, Fig. 8.13] is plotted along with the energy contained within a cylinder of radius a about the line focus as given by Eq. (14). We see that whereas more than 90% of all the energy in the focal plane is contained within the second dark ring of the Airy disk pattern, the energy associated with the line focus is not concentrated so much near the focal line. Every annular region of differential radius dr out perpendicular from the line focus contains the same amount of energy given from Eq. (15) by

$$d\epsilon \sim 2\sqrt{\pi}\,\frac{aI_0}{k}\,dr. \tag{16}$$

So the energy is distributed over all space in this manner.

The total energy over the surface of the reflector R is given by

$$E_R = 2\pi a \int_{-\infty}^{\infty} I(\vec{x})\,dy, \tag{17}$$



(a) Line focus

(b) Point focus [10, Fig. 8.13]

Fig. 3 — Comparison of the energy within a region of radius a from the center of focus for two fields

where we neglect the angle the surface of R makes with respect to the y axis and assume that the reflector is parallel to a cylinder of radius a normal to the y axis. By substituting Eq. (11) into (17) and using a tabulated integral [12, Eq. (860.110], we find that

$$E_R = 2(\pi)^{3/2} \sigma a \, I_0 \, J_0^2(ka). \tag{18}$$

By making use of the asymptotic approximation to $J_o(ka)$ for large ka [11, Eq. 9.2.1)], we have

$$E_R \underset{ka \to \infty}{\sim} \frac{2\sigma\lambda}{\sqrt{\pi}} I_0 \cos^2\left(ka - \frac{\pi}{4}\right). \tag{19}$$

We may now calculate the total energy output from the laser. The energy given by Eq. (19) is that contained in the standing wave inside of the laser resonator at the surface of the back reflector. The traveling waves each carry half the peak energy in the interference pattern represented by Eq. (19). Thus the power carried by the traveling wave inside of the resonator is just

$$p_i = c \frac{\sigma\lambda}{\sqrt{\pi}} I_0 , \tag{20}$$

where c is the speed of light. If we let α represent the coupling parameter defined by

$$\alpha \equiv \frac{p_o}{p_i} , \tag{21}$$

where p_o is the power in the output beam and p_i is the power in the traveling wave inside the resonator, then from Eq. (20) the output power from the laser is just

$$p_o = \alpha p_i = \frac{\alpha c \sigma \lambda I_0}{\sqrt{\pi}} . \tag{22}$$

If we assume that reasonable values for the various parameters in Eq. (22) are $cI_0 = 10^9$ W/cm^2, $\sigma = 1$ cm, $\lambda = 3.8 \times 10^{-4}$ cm, and $\alpha = 0.75$, we find that for gas breakdown at 10^9 W/cm^2 light intensity the output beam of a DF laser operating at atmospheric pressure can carry no more than 160 kW of power*.

*The breakdown threshold for 10.6-μm radiation in air is of the order of 10^9 W/cm^2, as given in Ref. 13. For lack of data we have assumed that this result is also approximately correct for breakdown over the DF and HF laser wavelengths.

CONCLUSIONS AND RECOMMENDATIONS

The HSURIA resonator as shown in Fig. 1 appears to be unsuitable for high-power lasers unless gas breakdown can be prevented somehow. The simplest way to solve the problem presently appears to be operation of the laser in vacuum. Where this is not practical, the resonator will have to be modified to eliminate the conical back reflector. Several modified resonators have been proposed which would do this. For example, one could replace the back reflector by a second reflaxicon and could add a simple back reflector or corner cube to return the beam back into the reflaxicon. Instead of adding the simple back reflector, one could bring the beam from the reflaxicon around into the other reflaxicon to form a ring resonator. Alignment of two reflaxicons, however, is a severe problem, but is is not clear that it can be avoided.

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Appendix

DERIVATION OF EQUATION (9)

In this appendix a proof is derived for the integral formula

$$\int_0^{\pi/2} \cos(kx \cos\theta) \cos(kz \sin\theta)\, d\theta = \frac{\pi}{2} J_0(k\sqrt{x^2+z^2}). \tag{A1}$$

We expand the two cosine factors in the kernel using the formulas*

$$\cos(dx \cos\theta) = J_0(kx) + 2 \sum_{n=1}^{\infty} (-1)^n J_{2n}(kx) \cos(2n\theta) \tag{A2}$$

and

$$\cos(kz \sin\theta) = J_0(kz) + 2 \sum_{\ell=1}^{\infty} J_{2\ell}(kz) \cos(2\ell\theta) \tag{A3}$$

and interchange the order of summation and integration to obtain an infinite series of terms. The first term is

$$\int_0^{\pi/2} J_0(kx)\, J_0(kz)\, d\theta = \frac{\pi}{2} J_0(kx)\, J_0(kz), \tag{A4}$$

*M. Abramowitz and L.A. Stegun, *Handbook of Mathematical Functions*, National Bureau of Standards, Washington, D.C., 1964, Eqs. (9.1.41) and (9.1.44).

in which the integration is trivial. The higher order terms ($n \geq 1$, $\ell \geq 1$) are of the form

$$4(-1)^n J_{2n}(kx) J_{2\ell}(kz) \int_0^{\pi/2} \cos(2n\theta)\cos(2\ell\theta)\, d\theta = 0, \text{ if } n \neq \ell, \tag{A5}$$
$$= (-1)^n \pi J_{2n}(kx) J_{2n}(kz), \text{ if } n = \ell,$$

in which the integral is easily evaluated by making the change of variables $\theta' = 2\theta$ and using the orthogonality relation*

$$\int_0^{\pi} \cos(m\theta')\cos(n\theta')\, d\theta' = 0, \text{ if } m \neq n \tag{A6}$$
$$= \frac{\pi}{2}, \text{ if } m = n.$$

Thus from Eqs. (A4) and (A5) we have

$$\int_0^{\pi/2} \cos(kx\cos\theta)\cos(kz\sin\theta)\, d\theta = \frac{\pi}{2}\left[J_0(kx) J_0(kz) + 2\sum_{n=1}^{\infty} (-1)^n J_{2n}(kx) J_{2n}(kz)\right]. \tag{A7}$$

The right-hand side of Eq. (7A) has the form of the right-hand side of the "summation theorem"†:

$$J_0(k\sqrt{x^2+z^2}) = J_0(kx) J_0(kz) + 2\sum_{n=1}^{\infty} (-1)^n J_{2n}(kx) J_{2n}(kz). \tag{A8}$$

By substituting Eq. (A8) into (A7), we obtain (A1), and the proof is complete.

*H.B. Dwight, *Tables of Integrals*, McMillan, New York, 1961, Eq. (858.517).

†I.S. Gradshteyn and I.M. Ryzhik, *Table of Integrals, Series, and Products*, 4th edition, Academic Press, New York, 1965, Eq. (8.531-1), in which we set $\phi = \pi/2$, $p = kx$, $r = kz$, and $m = 1$.